Multi-Resolution Projection vs. Low Pass Filtering in Large Eddy Simulation

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In Large Eddy Simulation (LES) equations are formally derived by applying a low-pass filter to the Navier-Stokes equations under the assumption that the differentiation and filtering operations commute. Commutation is generally satisfied if the filter has a constant width. However, this assumption is invalid if the filter width is not uniform—as in the case of wall-bounded flows—unless special filter operators are constructed. Recently a new class of *commutative* filters for both structured [1] and unstructured [2-3] grids has been developed. With these filters the differentiation and filtering operations commute to an *a priori* specified order of filter width.

The filtered convective term $\overline{u_i u_j}$ is unknown in LES and is typically decomposed into the convective term $\overline{u}_i\overline{u}_j$ that can be computed and the remainder, sub-grid scale (SGS) stress, which should be modelled:

$$\overline{u_i u_j} = \overline{u}_i\overline{u}_j - \underbrace{\left(\overline{u}_i\overline{u}_j - \overline{u_i u_j}\right)}_{\tau_{ij}}. \tag{1}$$

However, this formulation is inconsistent since the non-linear product $\overline{u}_i\overline{u}_j$ generates frequencies beyond the characteristic frequency that defines $\overline{\boldsymbol{u}}$. These high frequencies alias back as resolved ones and therefore act as fictitious stresses. In principle the subgrid-scale model, τ_{ij}, could exactly cancel this effect, but it is unlikely that such a model could be arranged. The obvious way to control the frequency content of the non-linear terms is to filter them. This strategy would result in the following alternative decomposition:

$$\overline{u_i u_j} = \overline{\overline{u}_i\overline{u}_j} - \underbrace{\left(\overline{\overline{u}_i\overline{u}_j} - \overline{u_i u_j}\right)}_{\overline{\tau}_{ij}}. \tag{2}$$

When this relation, together with a subgrid-scale model for $\overline{\tau}_{ij}$, is used one obtains a closed equation for $\overline{\boldsymbol{u}}$, but with an additional *explicit* filtering operation applied to the convective term. A possible drawback of this formulation is that the resulting LES equations are not Galilean invariant. The non-invariance comes from the fact that, in general, $\overline{\overline{\boldsymbol{u}}} \neq \overline{\boldsymbol{u}}$ since the low-pass filtering operation is not a projection operation.

In this talk we propose to use multi-resolution projection based on second generation wavelets [4] that are constructed in a spatial domain and can be customized for complex multi-dimensional domains and irregular sampling intervals. The wavelet based projection, $\mathcal{P}$, has the same commutation properties as commutative filters [1-3], but satisfy the following property: $\mathcal{P}^2\boldsymbol{u} = \mathcal{P}\boldsymbol{u}$. Consequently, with the wavelet multi-resolution projection an alternative decomposition (2) can be adopted. The details of the new formulation are discussed and the results for the case of turbulent channel flow are presented.

References

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